Stops

Sequenced Teaching of Problem Solving

HOME LEARNING MATERIAL

Year 6

Introduction

The modern maths curriculum in schools places a great focus on children's ability to solve problems and reason mathematically. When learning maths, children must be able to apply the core skills they have learned to a variety of problems and challenges.

At STOPS, we have devised 8 key problem-solving strategies that will help children approach problems with confidence. For each strategy, we have a range of problems that increase in difficulty so that children learn to tackle any tough maths problems with confidence.

Your child uses the STOPS problems in school and this book is designed to support and supplement the work that they are doing in school.

The problems and guidance in this book are designed to run in parallel to the work that your child is doing in school, enabling you to support them to become great mathematicians and successful problem-solvers - ensuring them success all the way to the end of primary school.

The STOPS Problem Solving Strategies:

A. Act it out / make a model

A great way to start solving problems. Act out, make or draw what the problem shows and you will be well on the way to solving it.

B. Trial and Error

A strategy every child must have - simply make some guesses and see how they go. Much better than not knowing how to start.

C. Trial By Improvement

The next step. Make an estimate, get a solution. Is it correct? Why not? How can we change our estimate to improve it? Children become more systematic.

D. Make a list or table

Many problems can be tackled by making a list of potential solutions. This can go hand-in-hand with strategies B and C to give children serious mental tools with which to solve tricky problems. Later, turn your list into organised tables and you can solve anything.

E. Find the pattern

Many problems can be solved by identifying a repeating pattern in shapes or numbers and using it to predict what may happen in other situations.

F. Simplify the problem

Some problems can be intimidating for children, but by making it more simple, it becomes more accessible.

G. Work Backwards

Start at the end and work back. Children will refine their skills of reasoning and 'inverse operations' to work their way through maths problems with ease.

H. Solve algebraically

It sounds more difficult than is, especially to children. When broken down into manageable steps of learning using shapes, symbols and eventually letters, children will become confident and experts in using algebra to solve problems

Using this booklet and how to best help your child

Each strategy has a one main problem to work through with your child and one other supporting problem. There are different steps within each strategy that make sure the problems are age-appropriate for your child. Remember that problem solving skills are very different to maths skills and children can develop at very different rates.

Each problem has notes afterwards that will give you guidance and examples of questions or modifications to support them if they are not sure or questions that could extend them if they are finding it easy.

Each problem is based on the STOPS problem-solving grid, where each strategy has 7 steps of difficulty for each of the 8 strategies. At the top of each problem in this book is the ‘step’ that the problem comes from, so that you can pick up on the next step of the strategy in every school year.

Some general tips:

- Encourage your child to make mistakes and feel positive about them, this is the only way to learn.
- Encourage children to record their thoughts in writing, on paper or in a special ‘problem solving’ notebook.
- Allow children time to think through for themselves, do not be tempted to do too much for them.

How do ‘steps’ and year groups work?

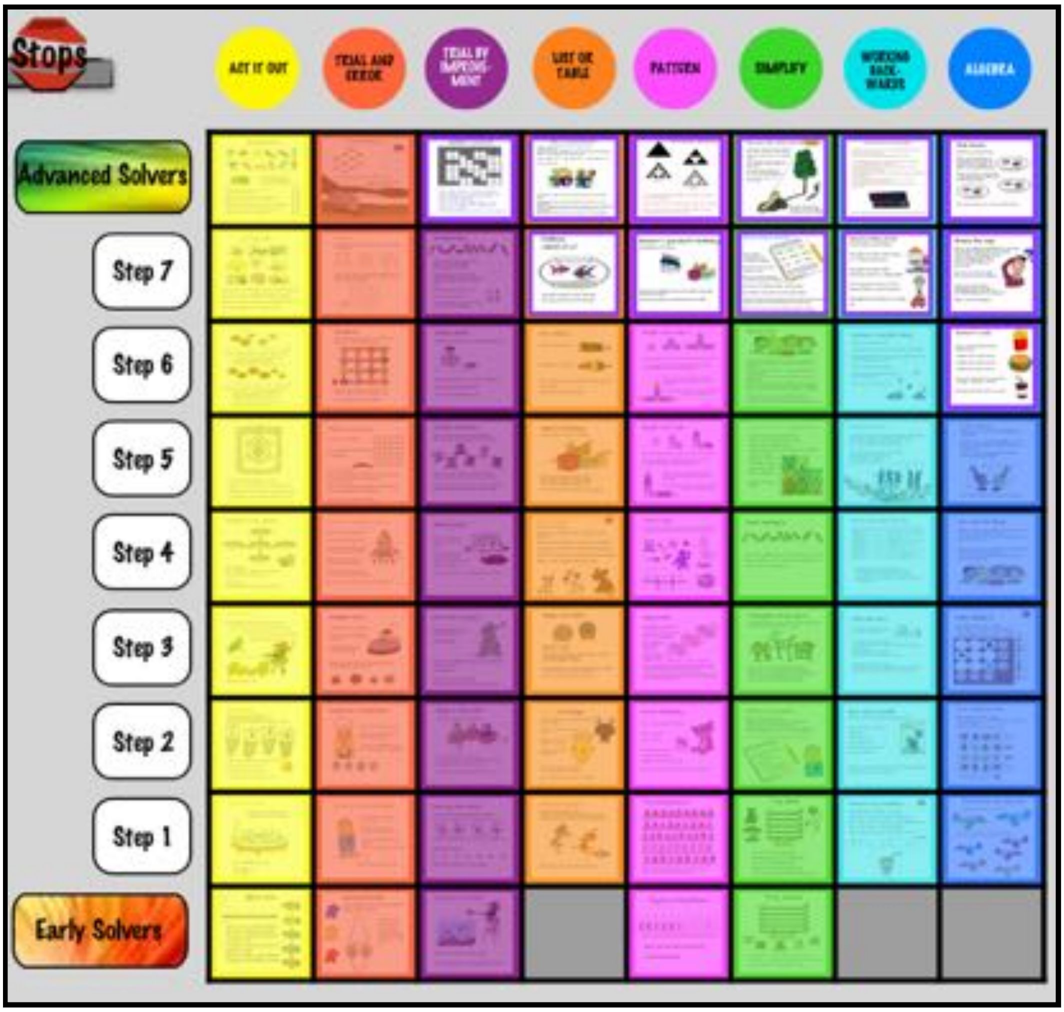
The STOPS problem solving skills are based on our famous grid, where each of the strategies has 7 steps within it that must be completed to be an expert problem-solver.

Each strategy is different, so ‘step 1’ does not always mean ‘year 1’.

Below is our grid, with the recommended year 5 problems highlighted. This book will provide support and companion problems to the year 5 set of problems that your child is studying at school.

You will notice that at year 6, the strategies ‘Act It Out / Make a Model’ , ’Trial and Error’ and ‘Trial By Improvement’ are now complete. Children should now be confident with these strategies and could be able to try some of our ‘advanced solvers’ problems in school.

This parent guide will begin at strategy D - “Make a List or Table”, as this will best support what your child is learning at school.



STRATEGY D – Make a List or Table

Step 7 – Hit the target

Luca is playing a game. He throws 10 balls at the target. They all hit the target and score points.

He is trying to make a total of exactly 150.

How could he make 150?

How to help:

- *Discuss the best method to start off with - should we just guess at random combinations or can we be more systematic?*
- *Make lists of the multiples of 18, 14, 12 and 10 - the solution must be from numbers in this list.*
- *When the list is created, we can use trial-by-improvement to make trials and adjust until a solution is found.*
- *If children find it hard, remove the 18 and 14 from the game. They could also score 12 and 8. Look for a total of 60 with 6 balls (3 x 12 and 3 x 8).*
- *If children find it difficult, ask them to find more than one solution.*

Solutions:

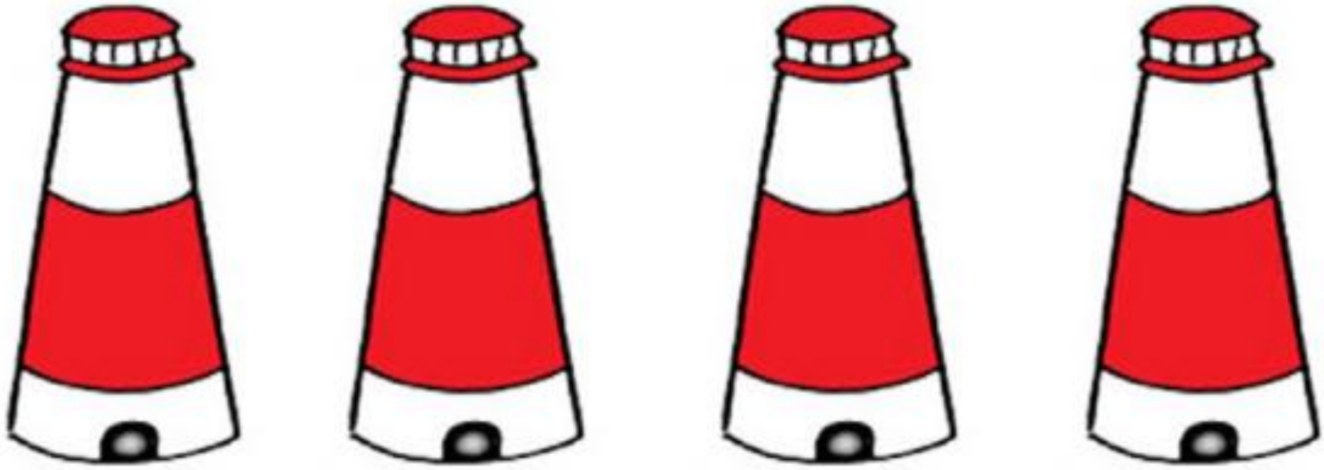
Several solutions are possible, including:

4 x 18 (72), 3 x 14 (42), 3 x 12 (36),

5 x 18 (90), 2 x 14 (28), 2 x 12 (24), 1 x 8 (8)

7 x 18 (126), 3 x 8 (24)

Step 7 – Lots of Lighthouses



There are four lighthouses on the coast. At first, all four lights come on together.

- The first lighthouse lights up for 3 seconds, then is off for 3 seconds.
- The second lighthouse lights up for 4 seconds, then is off for 4 seconds.
- The third lighthouse lights up for 6 seconds, then is off for 6 seconds.

The fourth lighthouse lights up for 7 seconds, then is off for 7 seconds.

When is the first time that all four lights will be off?

When is the next time that all four lights are on at the same time?

How to help:

- *Discuss the best method to set this out and help your child construct a table to show when each lighthouse is on and off, the full table that you need is below:*

Seconds	1	2	3	4
1	on	on	on	on
2	on	on	on	on
3	on	on	on	on
4	off	on	on	on
5	off	off	on	on
6	off	off	off	on
7	on	off	off	on
8	on	off	off	off
9	on	on	off	off
10	off	on	off	off
11	off	on	on	off
12	off	on	on	off
13	on	off	on	off
14	on	off	on	off
15	on	off	on	on
16	off	off	off	on
17	off	on	off	on
18	off	on	off	on
19	on	on	off	on
20	on	on	off	on
21	on	off	on	on
22	off	off	on	off
23	off	off	on	off
24	off	off	on	off
25	on	on	on	off
26	on	on	off	off
27	on	on	off	off

Seconds	1	2	3	4
28	off	on	off	off
29	off	off	off	on
30	off	off	off	on
31	on	off	on	on
32	on	off	on	on
33	on	on	on	on
34	off	on	on	on
35	off	on	on	on
36	off	on	off	off
37	on	off	off	off
38	on	off	off	off
39	on	off	off	off
40	off	off	off	off
41	off	on	on	off
42	off	on	on	off
43	on	on	on	on
44	on	on	on	on
45	on	off	on	on
46	off	off	off	on
47	off	off	off	on
48	off	off	off	on
49	on	on	off	on
50	on	on	off	off
51	on	on	on	off
52	off	on	on	off
53	off	off	on	off
54	off	off	on	off

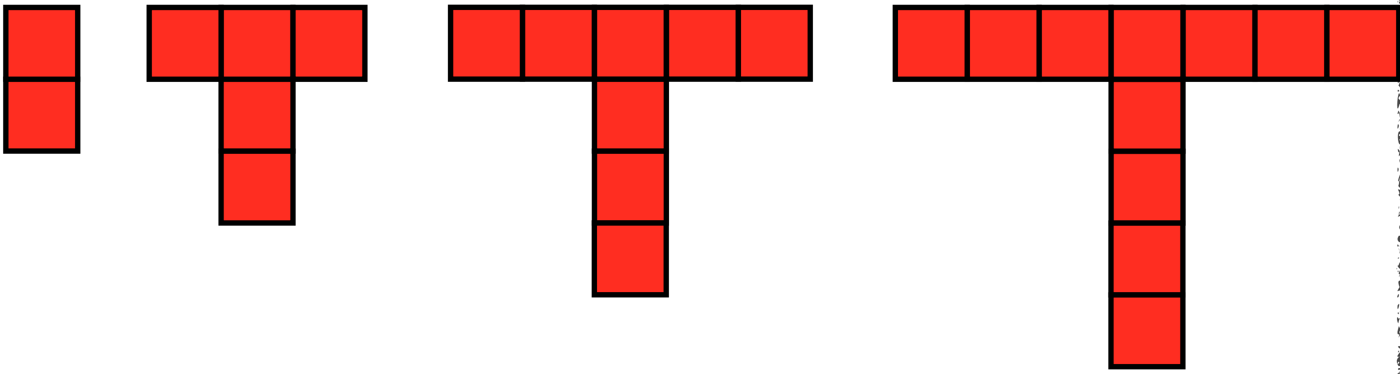
If children find this hard, just look at two of the lighthouses, for example the 3 and the 5 second lighthouses.
If children find this easy, extend to see when is the next time they will be off together and on together.

Solution: All four will be off after 40 seconds. All four will be on after 44 seconds.

STRATEGY E – Find the pattern

Step 7 - T-shape patterns

Josh makes these T-shape patterns with red tiles:



How many tiles does he add each time?

How many tiles will be in the next T-shape?

How many tiles will be in the 20th pattern?

Can you write an expression that shows the relationship between the number in the sequence and the number of tiles in the shape?

Use a table to help you

Number in sequence	1	2	3	4
Number of tiles	2	5	8	11

Use this expression to find out how many tiles will be in the 75th pattern in the sequence?

How to help:

- *First, establish that the sequence is increasing by 3 each time.*
- *We can then use this to extend the sequence by counting up in 3's.*
- *In year 6, children are expected to find an expression that links the two sequences. In this case, we multiply the number in the sequence by 3 and subtract one.*
- *We find this by seeing that the increase is always 3, so we can multiply by 3, then deciding how many to add or subtract. Trial and error will help you arrive at this answer.*
- *If your child is confident, this can be written algebraically:*

Number of tiles = (number in the sequence x 3) - 1

or

t = (3 x n) -1

or, if your child is confident with year 6 algebraic notation:

t = 3n -1

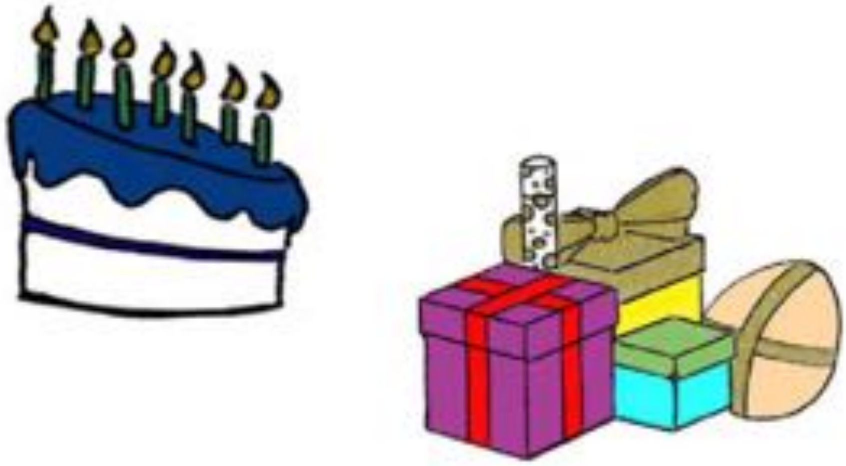
- *We can then use the formula to calculate the number of tiles needed for any shape in the sequence.*
- *Substitute 20 for n to get the solution for 20 tiles, e.g. (20 x 3) - 1 = 60 - 1 = 59.*
- *If you child finds it hard, ignore the algebraic notation*
- *If they find it easy, ask them to find how what number in the sequence has 65 tiles (solution: pattern number 22)*

Solution*:*

There are 59 tiles in the 20th pattern in the sequence.

There are 224 tiles in the 75th pattern in the sequence.

Step 7 – Birthday Cake



Every year, Aisha has a birthday cake and blows out candles. There are always the same amount of candles as her age.

When she was 7 years old, how many candles had she blown out in her life?

When she is 20, how many candles will she have blown out in total?

How to help:

- *First, start with some simple examples, e.g., how many candles will she have blown out at age 1, 2 and 3 etc.*
- *Use a table to record what you have found:*

Age	*Total number of candles*
1	*1*
2	*2 + 1 = 3*
3	*3 + 2 + 1 = 6*
4	*4 + 3 + 2 + 1 = 10*

- *Keep extending the sequence in this way until you reach 7 years.*
- *Discuss with your child what patterns they notice in the numbers. Pay attention to the difference between each one.*
- *The difference goes up by one each time: 1, 3, 6, 10, 15, 21, 28 etc. (These are known as the triangular numbers)*
- *We can use this sequence to extend up to 20 years, without having to add 20 + 19 + 18 … etc.*
- *If children find it hard, reduce the age from 20*
- *If children find it easy, increase the age to 40.*

Solution:

The sequence of triangular numbers increases in this way:

0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120,136, 153, 171, 190, 210

Aisha will have blown out 210 candles in total on her 20th birthday.

STRATEGY F – Simplify the problem

Step 7 – Beads and Necklaces

Aisha is making jewellery to sell. She is making necklaces and bracelets.

Each bracelet has 27 beads. She makes 68 bracelets.

Each necklace has 53 beads. She makes 29 necklaces.

She has bought 3200 beads. How many more does she need?

How to help:

- *Children will need good strategies for written and mental multiplication. Practice some 'long multiplication' first to make sure that they will not get confused during this multi-step task.*

Examples could include: 27 x 12, 35 x 43, 236 x 28.

- *It is very important that children organise their working-out carefully.*
- *Simplify the problem by breaking it down into steps.*
- *First, calculate the number of beads on the bracelets: 27 x 68.*
- *Label each stage clearly in their working-out.*
- *Next, calculate the number of beads on the necklaces: 53 x 29*
- *Add these totals to get the total number of beads used.*
- *We can use subtraction to find the difference between the total that she needs and the 3200 that she has bought.*
- *If children find it hard, change the problem so that there are single-digit numbers of beads on the necklace and the bracelet (for example, each bracelet has 5 beads, each necklace has 9 beads)*
- *If children find it easy, increase the number of beads and necklaces made to 3-digit numbers*

Solution:

She uses 1836 beads on the bracelets.
She uses 1537 beads on the necklaces.
This is a total of 3373 beads.
She needs to buy 173 more beads.

Step 7 – Jam Jars

Aisha is also making raspberry jam for the school fair.

Raspberries cost £3.40 per kilo
Sugar costs 99p per kilo
Glass jars cost 75p each.

She uses 15kg of raspberries and 10kg of sugar to make 20 full jars of jam.

Calculate the total cost to make 20 jars of jam?

If she sells all the jars at £4.50 per jar, how much profit does she make?

How to help:

- *This is a problem with many steps so it is very important that jottings are organised and labelled clearly.*
- *There are a lot of multiplication strategies in the problem. You could revise long multiplication (eg £17 x15, 75p x 12) as well as mentally multiplying decimals by 10 (eg 0.45 x 10 = 4.5, 32p x 10 = £3.20)*
- *Simplify the problem by breaking it down, step-by-step.*
- *First calculate the cost of the raspberries: £3.40 x 15. Long multiplication would work here, or you could 'partition' by multiplying £3.40 by 10 and by 5, then adding the two totals. Ask your child what method they would use and if they have other methods of doing the same calculation.*
- *Next, calculate the cost of the sugar: 99p x 10. Ask your child what methods they know for multiplying by 10. They should be moving the digits one place to the left and not resorting to a written method like long multiplication.*
- *Make sure jottings are clearly labelled and neatly set out.*
- *Now, calculate the cost of the glass jars: 75p x 20. Your child may know to multiply by 10 and then by 2 as a mental method or may resort to a written method.*
- *Now, we can calculate the overall cost of the 20 jars of jam by adding our three totals.*
- *Finally, multiply £4.50 by 20 (using a mental strategy if possible, multiply by 10 and then by 2) to see how much money she makes.*
- *Subtract the cost of the jam from the sales and see if Aisha has turned a profit!*
- *If children find this hard, change the cost of the raspberries to £3 and the sugar to £1.*
- *If children find this easy, repeat for higher amounts, e.g. 100 jars of jam, or even 1000!*

Solution:

The cost of the raspberries is £51
The cost of the sugar is £9.90
The cost of the jars is £15
The total cost of the 20 jars of jam is £75.90

Aisha will sell all 20 jars for £90, giving a profit of £14.10

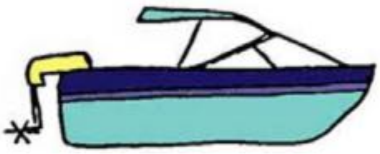
STRATEGY G – Working Backwards

Step 7 – Millionaire!

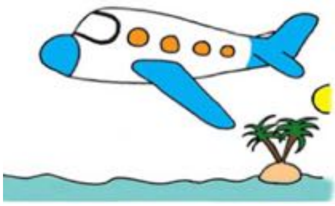
Roman's family won some money on the lottery.

They spent three-quarters of the money on a new house.

They spent three-quarters of the remaining amount on a sports car.



They then spent three-quarters of what was left on a boat.



They spent the last 20 000 on a luxury holiday.

How much did they win?

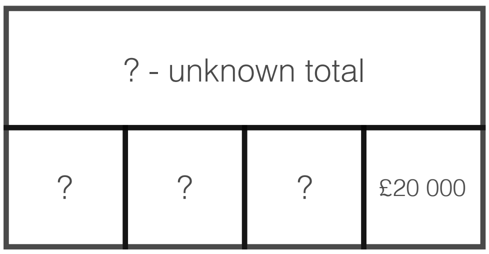
How to help:

- *Children will need a good working knowledge of fractions. Try setting problems like this to begin with:*

What is two-thirds of 423?

If two-thirds of a number is 90, what was the number? (Divide by 2 and times by 3 to get the answer - 135)

- *Read through the clues of the problem above.*
- *Model to children how to work backwards from the £20 000 total at the end.*
- *A bar model would be useful to model how this works:*



In the model above, we know that if he spent $^{3}/_{4}$, then $^{1}/_{4}$ must be left, therefore £20 000 is $^{3}/_{4}$ of the total. We can simply multiply by 4 to get the total amount.

- *Repeat this for each stage of the problem, working backwards through it. Make jottings along the way to record what each luxury item would have cost - this will help us with checking.*
- *When the original amount is found, work through the problem forwards to check by finding $^{3}/_{4}$ of each amount and checking that you end up with £20 000.*
- *If children find it hard, reduce the £20 000 to £200 or even £20.*
- *If children find it easy, change the £20 000 to a more complicated amount, e.g. £35 750.*

Solution:

Roman's family won £1, 280, 000 originally.

Step 7 – The Hockey Team

Aisha, Roman and Josh play in a hockey team.

Last season, they scored 61 goals between them.

This season, they did much better.

- Josh scored 3 times as many goals.
- Aisha scored 4 times as many goals.
- Roman scored 7 times as many goals.

This season, they all scored exactly the same number of goals.

How many goals did they each score in the first season?

How to help:

- *This may seem like a trial-and-error problem, and it can be solved that way. As always, allow you child time to think and tackle the problem as they see fit.*
- *Remind them that this is a working backwards strategy and to start with information that we know. We know that they all scored the same in the second season.*
- *We need to know how many they all scored in the first game. Whatever these numbers are, they must multiply by 3, 4 and 7 respectively to make the same number.*
- *A key piece of information is that the number of goals they scored in the second season must be divisible by 3, 4 and 7.*
- *We refer to this as the Lowest Common Multiple of 3, 4 and 7. Allow children some time to calculate this. We can find it by multiplying the three numbers together, although sometimes this does not give you the lowest.*
- *In this case, the LCM is 84.*
- *From here, we can work backwards:*

Josh: He scored 3 times as many, so what number multiplied by 3 equals 84? Use division to find out.

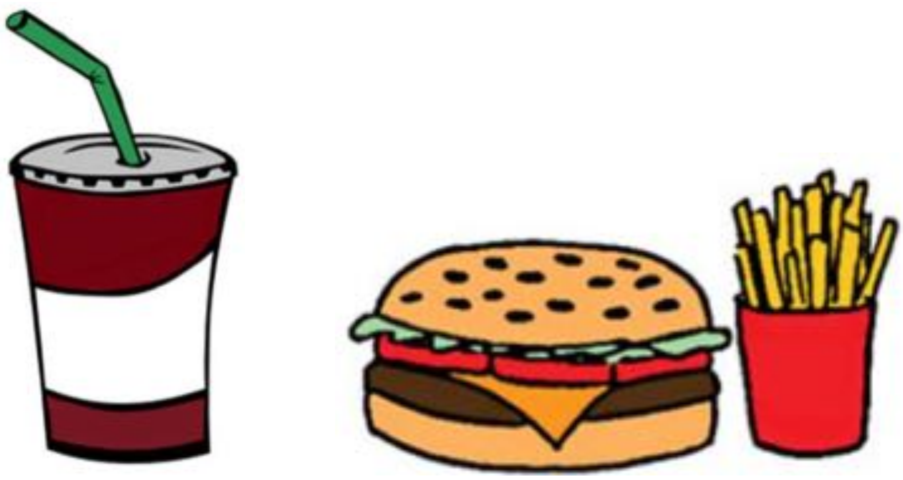
- *Work through the other two team-mates to find their totals for the first season. Check that they total 61.*
- *If children find it hard, generate the LCM for them.*
- *If children find it easy, see if they can make a similar puzzle for you to solve.*

Solution:

Josh scored 28.
Aisha scored 21.
Roman scored 12.

STRATEGY H – Solve Algebraically

Step 6 – Burger and Chips



These are some prices at the cafe:

Burger and a drink	–	£5
2 burgers and 2 chips	–	£11.50
chips + 2 drinks	–	£4.50

How much does each item cost on its own?

Write an expression to show the cost of a drink, chips and a burger, with the cost.

How to help:

- *At year 6, children should be learning how to express situations mathematically and solve simple equations.*
- *First, ask the child to think of how these prices could be written using letters, i.e. algebraically.*
- *Encourage them to use correct notation, e.g. instead of 1b + 1d = £5 it should read b + d = £5*
- *Begin with 'burger + chips = £5.00' if this is easier to understand, then replacing with letters.*
- *Express the other two prices as formulae:*

    2b + 2c = £11.50

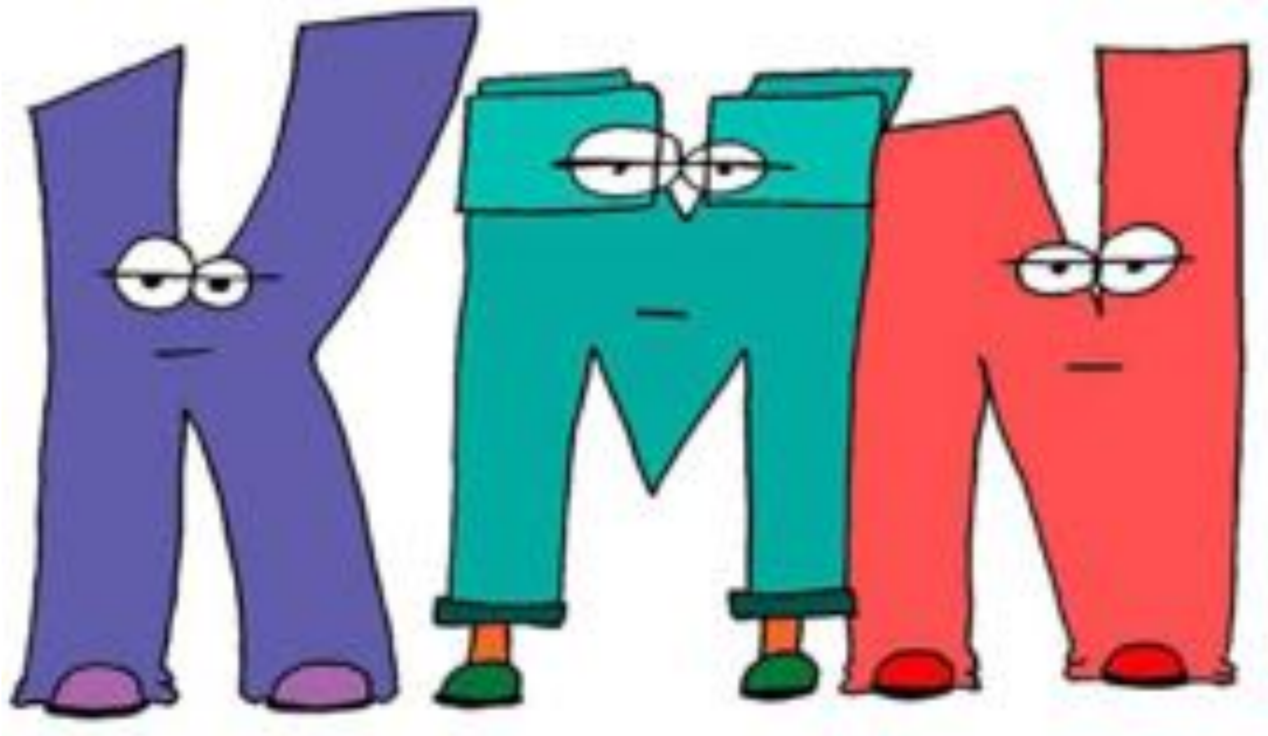
    c + 2d = £4.50.

- *It is way beyond the year 6 curriculum to solve these problems purely with algebra from this step, for example by using simultaneous equations.*
- *use a trial-by-improvement strategy from here. Pick trial values for b and d that total £5 and substitute those into the other equations to see if they fit.*
- *Record unsuccessful trials in a table.*
- *if children find it hard, use blank spaces or numbers instead of letters. Give them the cost of a drink to get them started.*
- *If children find it easy, set them other price combinations and ask them to create and solve the algebraic expressions to show the prices, e.g. 10 drinks, 12 chips and 15 burgers.*

Solutions:

A burger is £4.25
Chips are £1.50
A drink is 75p

Step 6 – k, m and n



k, m and n each represent a different whole number.

m is four times as big as n.
k is twice as big as n.
k, m and n add together to make a total of 2100.

Find out what k, m and n each represent.

How to help:

- *This problem is design for children to be able to express problems in terms of algebra.*
- *Start by looking at what expressions we can write down in algebraic form, for example:*

k + m + n = 2100

m = 4 x n   or   m = 4n

k = 2 x n   or   k = 2n

- *From here, we can substitute trial values for n and see if the final expression totals 2400.*

An example trial could be:
n = 100
m = 3n = 300
k = 2n = 200

100 + 300 + 200 = 600.

This does not make 2400 so we must adjust our trial and start again.

- *If children find it difficult, reduce 2100 to 21 and support them with creating and using the expressions.*
- *If children are confident, you could solve the problem by 'combining like terms.'.*

For example:

k + m + n = 2400

m = 4n   and   k = 2n

We can rewrite the first expression as: 2n + 4n + n = 2100

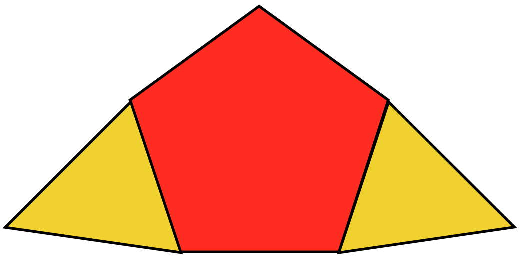
We can simplify this to: 7n = 2100, therefore n = 300.

Solution:

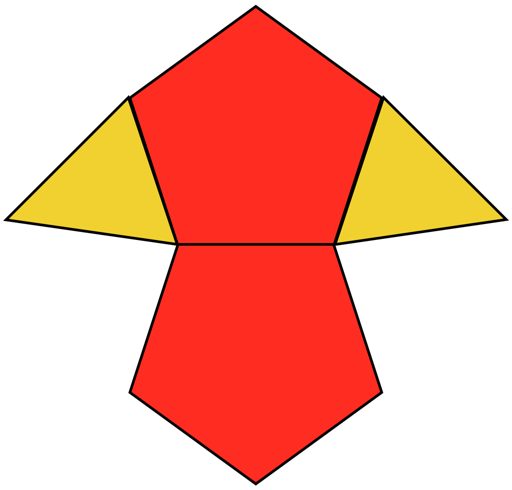
n = 300
m = 1200
k = 600

Step 7 – Shape problems.

Aisha is making new shapes with identical pentagons and triangles. She gives each shape a value.



Total value: 65



Total value: 102

Can you write two expressions for what she has made?

What is the value of the hexagon?

What is the value of the triangle?

What could be the next shape she makes and what would the value of it be?

How to help:

- *This problem also builds on skills learned in strategy E - Find The Pattern*
- *First, see how the shapes could be expressed in words or letters, for example.*

For the first shape:
One pentagon and two triangles = 65   or   h + 2t = 65

For the second shape:
Two pentagons and two triangles = 102   or   2h + 2t = 102

- *It is now not necessary at year 6 to solve these using algebra from this point.*
- *Instead, look for patterns. The second shape has one more hexagon than the first and the difference in the totals is 37. This means that the hexagon has a value of 37.*
- *We can now substitute 37 into one of the equations to find the solution, e.g.:*

h + 2t = 65
37 + 2t = 65
We can take the 37 away, leaving us with the total for two triangles:
2t = 28
t = 14

- *Encourage your child to check by substituting the values into the original equations.*
- *If children find it hard, use a trial and error strategy to find a solution. Give them the value of the hexagon if needed.*
- *If children find it easy, ask them to create more patterns with triangles and hexagons for them to write algebraically and solve.*

Solutions:

The triangle is worth 14.
The hexagon is worth 37.

Step 7 – Old and young

There are three pupils and a headteacher.
Two of the pupils are the same age.
The product of ages of the three pupils is 2535.
The sum of their ages is exactly the age of the headteacher, 41 years old.

How old are the three children?

How to help:

- *Your child will need to be familiar with the term 'product' for multiplication*
- *Encourage the children to write down algebraic expressions for the information in the problem, for example:*

Child A x Child B x Child C = 2535   or a x b x c = 2535

Two of the children are the same age, so a=b.   In this case a x a x c = 845, or a^2 x c = 2535

Child A + Child B + Child C = Headteacher   or   a + b + c = h

- *From this point, use a trial-by-improvement strategy to make trials of ages and record in a table, for example:*

a	*b*	*c*	*a^2 x c*	*a + b + c*
14	*14*	*15*	*2950*	*43*

- *If children find it hard, ignore the squared term and just focus on a x b x c. You could allow a calculator if they are not confident with the multiplication. This will enable them to focus on the problem solving skills.*
- *If children find it easy, remove the need for tow children to be at the same age. Pick three new ages and ask them to solve (eg the three children's ages have a product of 2520, solution: 12, 14 and 15 years old)*

Solution:

The children are 13, 13 and 15 years old.

Well done - you have solved all of the year 6 logic and reasoning problems in this booklet and you are an expert problem-solver!

Head over to www.stopsproblemsolving.co.uk and check out the COVID19 zone for more games and problems.